

Absorption of solar radiation by solar neutrinos

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Abstract. We calculate the absorption probability of photons radiated from the surface of the Sun by a left-handed neutrino with definite mass and a typical momentum for which we choose $|p_1| = 0.2 \text{ MeV}$, producing a heavier right-handed antineutrino. Considering the two transitions $\nu_1 \rightarrow \nu_2$ and $\nu_2 \rightarrow \nu_3$ we obtain the two oscillation lengths $L_{12} = 4960.8 \text{ m}$, $L_{23} = 198.4 \text{ m}$, the two absorption probabilities $P_{12}^{\text{abs.}} = 2.5 \times 10^{-67}$, $P_{23}^{\text{abs.}} = 1.2 \times 10^{-58}$ and the two absorption ranges $R_{12}^{\text{abs.}} = 4.47 \times 10^4 R_\odot = 208.0 \text{ au}$, $R_{23}^{\text{abs.}} = 0.89 \times 10^4 R_\odot = 41.4 \text{ au}$, using a neutrino mass differences of $\sqrt{|\Delta m_{12}^2|} = 10 \text{ meV}$, $\sqrt{|\Delta m_{23}^2|} = 50 \text{ meV}$ and associated transition dipole moments. We collect all necessary theoretical ingredients, i.e. neutrino mass and mixing scheme, induced electromagnetic transition dipole moments, quadratic charged lepton mass asymmetries and their interdependence.

The purpose of this paper is to determine the absorption probability of photons radiated from the surface of the Sun by a left-handed neutrino with definite mass, which produces a heavier right-handed antineutrino. To reach this goal, we start with the geometrical and thermal properties of surface radiation of the Sun to determine the photon flux. The elementary absorption cross section involves transition electric and magnetic dipole moments of neutrinos taken to be Majorana particles, in a minimal extension of the standard model to account for neutrino mass and mixing [1, 2]. The electromagnetic transition dipole moments are to be evaluated on the appropriate mass shells for the neutrino–antineutrino transition. As such, they are by definition gauge invariant and must be formed generally combining the vertex and the propagator Green functions [3].

Interestingly, in a recent paper [4], the authors consider the fate of an electron neutrino, undergoing the analogous transition in the interior of the Sun induced by the interior magnetic field. They couple this transition with normal oscillations to produce $\bar{\nu}_e$ from ν_e , to be subsequently observed on Earth.

To be definite, we consider the absorption cross section of a photon inducing an electromagnetic dipole transition from the initial definite mass eigenstate $\nu_{L,j}(p_1)$ with mass $m_{\nu_{L,j}} = m_j$, with left-handed helicity, thus denoted “neutrino”, to a right-handed mass eigenstate $\bar{\nu}_{R,k}(p_2)$ with mass $m_{\nu_{R,k}} = m_k$, denoted “antineutrino”. This yields the absorption probability per neutrino when integrated from the solar surface to infinity. The kinematics of the dipole transition is then generally the following:

$$\gamma(q) + \nu_{L,j}(p_1) \longrightarrow \bar{\nu}_{R,k}(p_2), \quad (1)$$

and $\Delta m_{jk} = m_k - m_j > 0$, with $j < k$ ($= 1, 2, 3$). We assume hierarchical masses: $m_1 \ll m_2 \ll m_3$ and to simplify this kinematics, we set $m_1 = 0$ throughout the paper.

The absorbing neutrino ($\nu_{L,j}$) is taken to emerge from the center of the Sun, defining the radial direction as shown in Fig. 1. From the geometry indicated in Fig. 1 we obtain the integral flux in the solar rest frame at the time when the neutrino $\nu_{L,j}(p_1)$ is at a distance d from the surface of the Sun:

$$\begin{aligned} \phi &= 2\pi R_\odot^2 \int_{1/a}^1 d \cos \Theta \\ &\times \int_0^\infty \frac{q^2 dq}{4\pi^3} \frac{n_K(T)}{\rho^2} \cos \alpha \cos \theta (1 - \cos \theta), \end{aligned} \quad (2)$$

where $a = 1 + d/R_\odot$, $\alpha = \Theta + \theta$, with $R_\odot \cos \Theta + \rho \cos \theta = R_\odot + d$ and $\rho \sin \theta = R_\odot \sin \Theta$. The above flux represents the Planckian surface radiation of the Sun with temperature T and occupation number $n_K(T)$, given by

$$n_K(T) = (e^{K/T} - 1)^{-1}.$$

The differential flux multiplied by the absorption cross section σ yields the absorption rate per unit time $\Gamma_d(\nu_{L,j} \gamma \rightarrow \bar{\nu}_{R,k})$:

$$\Gamma_d(\nu_{L,j} \gamma \rightarrow \bar{\nu}_{R,k}) = -\frac{d}{dt} \log P = \int \sigma d\phi, \quad (3)$$

where P represents the survival probability of the initial neutrino $\nu_{L,j}(p_1)$. For $c = 1$, we have $t = d$, starting the

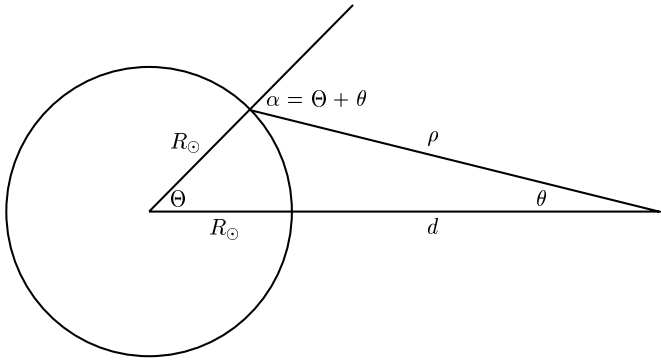


Fig. 1. Geometric properties underlying the absorption reaction in the reaction plane. The emerging left-handed neutrino is at a distance $R_\odot + d$ from the Sun's center, illuminated by photons emitted at an angle α relative to the normal of the emitting solar surface element. Thus, the triangle with the angles θ , Θ and α is formed

clock ($t = 0$) when the neutrino $\nu_{L,j}(p_1)$ crosses the surface. The integral survival probability P_∞ is thus given by

$$P_\infty = e^{-\int_0^\infty \Gamma_{d=t} dt} = 1 - P_{\text{abs.}}, \quad (4)$$

where $P_{\text{abs.}}$ is the total probability for the absorption of photons.

Next, we define the absorption probability within a distance d as

$$P_{\text{abs.}}(d) = 1 - e^{-\int_0^d \Gamma_{d'=t} dt} \simeq \int_0^d \Gamma_{d'=t} dt, \quad (5)$$

because it turns out, as expected, that $P_{\text{abs.}}(d) \leq P_{\text{abs.}} \ll 1$.

The absorption cross section σ is given by

$$\sigma_{jk} = 2\pi |\Delta m_{jk}^2| \frac{|\mu_{jk}|^2}{2p_1 q} \delta \left(\cos \theta - \left(1 - \frac{\Delta m_{jk}^2}{2p_1 q} \right) \right), \quad (6)$$

with $|\Delta m_{jk}^2| = |m_{\nu_k} - m_{\nu_j}|^2$, and $|\mu_{jk}|$ is the magnetic transition dipole moment of neutrino.

Because of the purely left-handed neutrino emitted by the Sun, there appears no spin average over neutrino helicities, contrary to photon polarizations.

The neutrino dipole moments are determined from the effective standard model photon–neutrino vertex $\Gamma_\mu^{\text{eff}}(\gamma\nu\bar{\nu})$ [5–13]. The transition $\nu_j \rightarrow \nu_k \gamma$ is an electroweak process induced at leading 1-loop order. This order involves the so-called “neutrino-penguin” diagrams through the exchange of $\ell = e, \mu, \tau$ leptons and weak bosons and is given by [3,6,9]

$$\begin{aligned} e^\mu(q) \Gamma_\mu^{\text{eff}}(\gamma\nu\bar{\nu}) &= e^\mu(q) \left[G_1(q^2) \bar{\nu}_k (\gamma_\mu q^2 - q_\mu \not{q}) \nu_{jL} \right. \\ &\quad \left. + i G_2(q^2) (m_{\nu_k} \bar{\nu}_k \sigma_{\mu\nu} q^\nu \nu_{jL} + m_{\nu_j} \bar{\nu}_k \sigma_{\mu\nu} q^\nu \nu_{jR}) \right]. \quad (7) \end{aligned}$$

The above vertex is invariant under the electromagnetic gauge transformations. The first term in (7) vanishes identically for a real photon due to the electromagnetic gauge condition. The expression (7) yields the electric and magnetic dipole moments [6,9]

$$d_{kj}^{\text{el}} = \frac{1}{2} (m_{\nu_j} - m_{\nu_k}) G_2(0)_{kj}, \quad (8)$$

$$\mu_{kj} = \frac{1}{2} (m_{\nu_j} + m_{\nu_k}) G_2(0)_{kj}, \quad (9)$$

$$G_2(0)_{kj} = \frac{2e}{M^{*2}} \sum_{\ell=e,\mu,\tau} U_{ki}^\dagger U_{ij} F(x_{\ell_i}), \quad x_{\ell_i} = \frac{m_{\ell_i}^2}{m_W^2}, \quad (10)$$

where $i, j, k = 1, 2, 3$ denotes the neutrino species, and

$$F(x_{\ell_i}) \simeq -\frac{3}{2} + \frac{3}{4} x_{\ell_i} \quad (11)$$

was obtained after the loop integration and for $x_{\ell_i} \ll 1$. Here $M^* = 4\pi v = 3.1 \text{ TeV}$, and $v = (\sqrt{2} G_F)^{-1/2} = 246 \text{ GeV}$ represents the vacuum expectation value of the scalar Higgs field.

Note that for the off-diagonal transition moments, the first term in (11) vanishes in the summation over ℓ due to the orthogonality condition of the mixing matrix U .

For the Majorana neutrinos considered here this matrix is approximately unitary and necessarily of the form

$$\sum_{i=1}^3 U_{ki}^\dagger U_{ij} = \delta_{kj} - \varepsilon_{kj}, \quad (12)$$

where ε is a hermitian nonnegative matrix (i.e. with all eigenvalues non-negative) and

$$\begin{aligned} |\varepsilon| &= \sqrt{\text{Tr } \varepsilon^2} = \mathcal{O}(m_{\nu_{\text{light}}}/m_{\nu_{\text{heavy}}}), \\ &\sim 10^{-22} \text{ to } 10^{-21}. \quad (13) \end{aligned}$$

The case $|\varepsilon| = 0$ is excluded by the very existence of oscillation effects. As a consequence, there is no exact GIM cancellation in lepton-flavour space, unlike for the cases of off-diagonal transition moments and the quark flavours.

In the Majorana case calculation of the “neutrino–antineutrino-penguin” diagrams, using charged lepton and an antilepton propagators in the loops, produces transition matrix elements which are complex antisymmetric quantities in lepton-flavour space. Finally,

$$d_{jk}^{\text{el}} = \frac{1}{2} (m_{\nu_j} - m_{\nu_k}) 2 \text{Re } G_2(0)_{kj}, \quad (14)$$

$$\mu_{jk} = \frac{1}{2} (m_{\nu_j} + m_{\nu_k}) 2i \text{Im } G_2(0)_{kj}. \quad (15)$$

Dipole moments describing the transition from Majorana neutrino mass eigenstate flavour ν_j to ν_k have the following form [2,3,5,8]:

$$d_{jk}^{\text{el}} = \frac{3e}{2M^{*2}} (m_{\nu_j} - m_{\nu_k}) \sum_{\ell=e,\mu,\tau} \frac{m_{\ell_i}^2}{m_W^2} \text{Re } U_{ki}^\dagger U_{ij}, \quad (16)$$

$$\mu_{jk} = \frac{3ei}{2M^{*2}} (m_{\nu_j} + m_{\nu_k}) \sum_{\ell=e,\mu,\tau} \frac{m_{\ell_i}^2}{m_W^2} \text{Im } U_{ki}^\dagger U_{ij}. \quad (17)$$

The quadratic charged lepton mass asymmetry is generated by the electric and magnetic dipole form factors only. The quantities U_{ij} incorporate the neutrino-flavour mixing matrix governing the decomposition of a coherently produced

left-handed neutrino $\tilde{\nu}_{L,\ell}$ associated with charged-lepton flavour $\ell = e, \mu, \tau$ into the mass eigenstates $\nu_{L,i}$:

$$|\tilde{\nu}_{L,\ell}; \mathbf{p}\rangle = \sum_i U_{\ell i} |\nu_{L,i}; \mathbf{p}, m_i\rangle. \quad (18)$$

We emphasise that the sensitivity of the dipole moments (16), (17) is much larger because of the τ -loop than in the oscillations where only the mixing angles and mass differences (-square), $|\Delta m_{13}^2| \simeq |\Delta m_{23}^2|$, enters.

We proceed to numerical evaluation of the transition dipole moments which in general receive very small contributions because of the smallness of the neutrino mass, $|m_\nu| \sim 10^{-2}$ eV. For the dipole moments the dominant contributions are coming from the $1 \rightarrow 2$ and $2 \rightarrow 3$ transitions. Since the mixing matrix element $|U_{1\tau}|$ is small, and today still unknown, in the evaluation of the $1 \rightarrow 2$ transition we assume the dominance of the μ -loops:

$$\left(\begin{array}{c} |d_{12}^{\text{el}}| \\ |\mu_{12}| \end{array} \right) \simeq \frac{3e}{2M^{*2}} \frac{m_\mu^2}{m_W^2} \sqrt{|\Delta m_{12}^2|} \left(\begin{array}{c} |\text{Re} U_{1\mu}^\dagger U_{\mu 2}| \\ |\text{Im} U_{1\mu}^\dagger U_{\mu 2}| \end{array} \right). \quad (19)$$

The dominant contributions to the electric and magnetic transition dipole moments of the neutrinos for the $2 \rightarrow 3$ transition are, due to the τ -loops, proportional to the Re and Im part of $U_{2\tau}^\dagger U_{3\tau}$ and given by

$$\left(\begin{array}{c} |d_{23}^{\text{el}}| \\ |\mu_{23}| \end{array} \right) \simeq \frac{3e}{2M^{*2}} \frac{m_\tau^2}{m_W^2} \sqrt{|\Delta m_{23}^2|} \left(\begin{array}{c} |\text{Re} U_{2\tau}^\dagger U_{\tau 3}| \\ |\text{Im} U_{2\tau}^\dagger U_{\tau 3}| \end{array} \right). \quad (20)$$

Setting for the matrix elements $|\text{Im} U_{1\mu}^\dagger U_{\mu 2}| \simeq 0.32$ and $|\text{Im} U_{2\tau}^\dagger U_{\tau 3}| \simeq 0.5$ [8, 10], and specifically for the hierarchical masses $\sqrt{|\Delta m_{12}^2|} \simeq 10$ meV [14] and $\sqrt{|\Delta m_{32}^2|} \simeq 50$ meV [15] and transforming the moments to Bohr magnetic units, we have found the following standard transition magnetic moments of neutrinos:

$$|\mu_{12}|_{\text{st}} \simeq 3.12 \times 10^{-34} \text{ eV}^{-1} = 1.05 \times 10^{-27} \mu_B, \quad (21)$$

$$|\mu_{23}|_{\text{st}} \simeq 6.14 \times 10^{-31} \text{ eV}^{-1} = 2.07 \times 10^{-24} \mu_B. \quad (22)$$

From the above equations we see that the transition $1 \rightarrow 3$ is sensitive to the value of $|U_{1\tau}|$ down to the $\mathcal{O}(10^{-3})$. Note finally that direct experimental evidence for neutrino-flavour transformation from neutral-current interactions (7) is given in [16].

The absorption rate per unit time in the solar rest system at a given distance d from the Sun's surface is

$$\Gamma_{jk}^d(\nu_{L,j} \gamma \rightarrow \bar{\nu}_{R,k}) = \frac{1}{2\pi^2} \int q^2 dq d\cos\theta n_K(T) \quad (23)$$

$$\times \sigma_{jk}(q, \theta) \cos\theta (1 - \cos\theta) \frac{\sin\Theta \cos(\theta + \Theta)}{\sin\theta(a \cos\Theta - 1)}.$$

In (23) the factor $\sin\Theta \cos(\theta + \Theta)/\sin\theta(a \cos\Theta - 1)$ is equal to 1, as a consequence of the geometry described in Fig. 1.

From (6), (21) and (22), we obtain the standardized form of the absorption cross section:

$$\sigma_{\text{st}} = X_{jk} \sigma_{jk}^{\text{red.}} = \frac{X_{jk}}{2p_1 q} \delta \left(\cos\theta - \left(1 - \frac{\Delta m_{jk}^2}{2p_1 q} \right) \right), \quad (24)$$

where $\sigma_{jk}^{\text{red.}}$ represents the reduced cross section and X_{jk} is the dimensionless quantity

$$X_{jk} = 2\pi |\Delta m_{jk}^2| |\mu_{jk}|_{\text{st}}^2, \quad (25)$$

which receives the following values for the respective magnetic moments (21) and (22) and the mass differences:

$$X_{12} = 6.10 \times 10^{-71}, \quad (26)$$

$$X_{23} = 5.91 \times 10^{-63}. \quad (27)$$

The reduced cross section $\sigma_{jk}^{\text{red.}}$ leads to the following reduced dimensionless absorption rate $\Gamma_{jk}^{\text{red.}}$:

$$\Gamma_{jk}^{\text{red.}}(a) = \frac{1}{2\pi^2} R_\odot \int q^2 dq d\cos\theta n_K(T)$$

$$\times \sigma_{jk}^{\text{red.}}(q, \theta) (1 - \cos\theta) \cos\theta. \quad (28)$$

The integral

$$f_{jk}(A) = \int_1^A da \Gamma_{jk}^{\text{red.}}(a), \quad A = 1 + \frac{d}{R_\odot} \quad (29)$$

determines the absorption probability defined by (5) within a distance d from the solar surface:

$$P_{jk}^{\text{abs.}}(d) = X_{jk} f_{jk}(A), \quad P_{jk}^{\text{abs.}} = X_{jk} f_{jk}(\infty). \quad (30)$$

The radius of the Sun given in different units is

$$R_\odot = 6.961 \times 10^8 \text{ m} = 2.322 \text{ s} = 3.528 \times 10^{15} \text{ eV}^{-1}. \quad (31)$$

The astronomical unit (Sun–Earth distance) corresponds to $\text{au} \equiv \oplus - \odot = 499.005 \text{ s}$, which means that $A_\oplus = \text{au}/R_\odot = 214.91$.

We choose $p_1 = 0.2 \text{ MeV}$ for the momentum of the solar neutrino emerging from the solar interior through the surface [17], yielding the oscillation lengths L_{12} and L_{23} of the neutrinos $\nu_1 \longleftrightarrow \nu_2$ and $\nu_2 \longleftrightarrow \nu_3$, respectively:

$$L_{12} = \frac{4\pi p_1}{|\Delta m_{12}^2|} = 25.133 \times 10^9 \text{ eV}^{-1} = 4960.8 \text{ m}, \quad (32)$$

$$L_{23} = \frac{4\pi p_1}{|\Delta m_{23}^2|} = 1.005 \times 10^9 \text{ eV}^{-1} = 198.4 \text{ m}. \quad (33)$$

The solar activity is characterized by the surface temperature T_\odot , which we take as $0.5 \text{ eV} = 5802.5 \text{ K}$. We note that a significant contribution to the absorption probability comes from wave-lengths of solar radiation comparable with the oscillation lengths L_{12} and L_{23} . Those long wave-lengths constitute the Rayleigh–Jeans tail of the Planckian spectrum. For the standards defined above, we present the functions $f_{12}(A)$ and $f_{23}(A)$ in Figs. 2 and 3, respectively. The enlarged scale figures are displayed separately up to $\simeq 1 \text{ au}$.

The analytic structure of the functions $f_{12}(A)$ and $f_{23}(A)$ gives rise to a characteristic range of solar activity in the absorption process, independently of the small absorption probability:

$$R_{12}^{\text{abs.}} = R_\odot \sqrt{\frac{2p_1 T_\odot}{|\Delta m_{12}^2|}} = 4.47 \times 10^4 R_\odot = 208.0 \text{ au}, \quad (34)$$

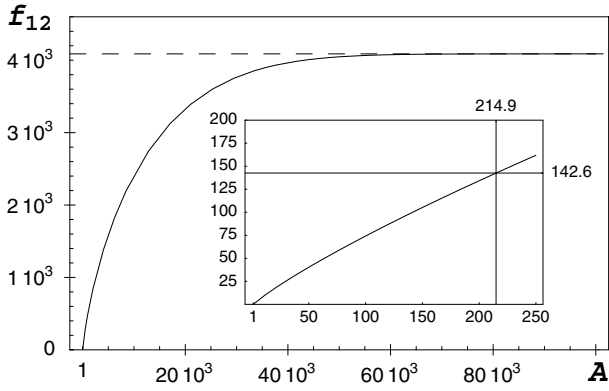


Fig. 2. The absorption probability $f_{12}(A)$ within a distance d from the surface of the Sun as a function of $A = 1 + d/R_{\odot}$

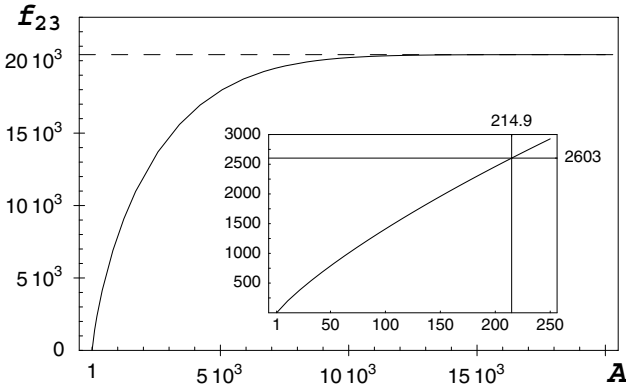


Fig. 3. The absorption probability $f_{23}(A)$ within a distance d from the surface of the Sun as a function of $A = 1 + d/R_{\odot}$

$$R_{23}^{\text{abs.}} = R_{\odot} \sqrt{\frac{2p_1 T_{\odot}}{|\Delta m_{23}^2|}} = 0.89 \times 10^4 R_{\odot} = 41.4 \text{ au.} \quad (35)$$

The absorption range $R_{23}^{\text{abs.}}$, for example corresponds to a distances of 2 astronomical units beyond the semi-major axis of the orbit of Pluto. The ranges $R_{12}^{\text{abs.}}$, $R_{23}^{\text{abs.}}$ are clearly visible near $A = 5 \times 10^4$ in Fig. 2, in the approach of the function $f_{12}(A)$ to its asymptotic value $f_{12}(\infty) = 4087.9$, and near $A = 10^4$ in Fig. 3, in the approach of the function $f_{23}(A)$ to its asymptotic value $f_{23}(\infty) = 20418.8$. Thus the total absorption probabilities per neutrino become

$$P_{12}^{\text{abs.}} = X_{12} f_{12}(\infty) \simeq 2.5 \times 10^{-67}, \quad (36)$$

$$P_{23}^{\text{abs.}} = X_{23} f_{23}(\infty) \simeq 1.2 \times 10^{-58}. \quad (37)$$

For the bound on the neutrino magnetic moment, $\mu_{\nu_e} \lesssim 3 \times 10^{-12} \mu_B$, derived from SN1987A [18], we would obtain $P_{\text{abs.}} \sim 10^{-35}$. Taking 10^{40} similar neutrinos, emitted from the pp cycle of the Sun, the rate of the antineutrinos thus produced from the Sun would be 10^5 per year. Our number (36) is meant as a lower bound.

In this work we collected all necessary theoretical ingredients, i.e. neutrino mass and mixing scheme, induced electromagnetic transition dipole moments, quadratic mass asymmetries entering the process of photon absorption by a definitive neutrino flavor producing a massive antineu-

trino. Obviously the same electromagnetic dipole transitions occur under many more circumstances, e.g. in the core of supernova explosions, in reactions with the cosmic microwave background radiation, in strong magnetic fields like in neutron stars, in coherent maser light directed at a nuclear reactor. These processes may well lead to larger effects.

As follows from the geometric aspects discussed in Figs. 1, 2 and 3 and displayed in (30)–(37), the absorption probability per neutrino scales from our solar example to another similar situation, modulo minor logarithmic correction factors, as

$$\frac{R \langle T \rangle}{p^2}, \quad (38)$$

where R_{\odot} is replaced by an irradiating photon surface of radius R , $\langle T \rangle$ denotes an average temperature of surface of light emission, replacing $T_{\odot} = 0.5 \text{ eV}$, and p denotes the momentum of the emitted neutrino. From the above scaling law it follows that no dramatic changes in the absorption probability (per neutrino or antineutrino) occur in a supernova explosion. There the momenta are changed to $\mathcal{O}(10\text{--}20 \text{ MeV})$ from the neutrinosphere, R is adapted to the size of the source before the neutrino is beyond the range of its radiation, i.e. cannot be many orders of magnitude larger than the solar radius, and $\langle T \rangle$ is (much) below the MeV scale, when becoming effective in the process of the $\nu \rightarrow \bar{\nu}$ dipole transition. The only difference is in the number of neutrinos emitted, offset by the distance (-square) attenuation, towards an observer on Earth. For this reason we refrain from giving numerical estimates for supernova explosions here.

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